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Acoustic Torques and Forces on Disks

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Abstract

The time-average forces and torques exerted by a plane sound wave upon fixed rigid disks of various shapes are calculated. Results are given for disks bounded by smooth closed convex curves. These results are then specialized to ellipses and circles. Results are also given for infinitely long thin strips. The results are all valid for ka large, where $k = \frac{2\pi}{\lambda}$, λ being the wavelength and a being a typical dimension of the disk. The oscillatory behavior of the torque as a function of ka and the occurrence of numerous equilibrium positions are interesting consequences of these calculations.

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1. Introduction

When a sound wave strikes an object it exerts a periodic force and torque on the object. In this paper we calculate the mean values of these quantities for certain objects which are thin, rigid and immobile. We call such objects disks. We will consider disks whose rims are smooth closed convex curves, and also disks which are infinitely long strips. The circular Rayleigh disk^[1] is included. Our results are based upon the 'geometrical theory of diffraction'^[2], a new method for solving diffraction problems. This method applies to situations in which the wavelength λ is small compared to the obstacle dimensions. However it also works well for wavelengths as large as the obstacle

This defines the range in which our present results apply; note that high-frequency sound is included in this range.

Most of the previous theoretical work on this subject has dealt with the low-frequency case, in which the wavelength is large compared to the obstacle dimensions. W. König^[3] calculated the torques on an ellipsoid and on a circular disk as well as the attractive or repulsive force between two spheres^[4] in the limit of infinite wavelength. M. Kotani^[5] extended the calculation of the torque on a circular disk to include several powers of λ^{-1} . L. V. King^[6] calculated the force on a sphere to several terms in λ^{-1} . He also calculated to the same order the force on a circular disk due to a normally incident plane wave, taking account of the motion of the disk^[7]. This same force was measured by R. W. Boyle and J. F. Leimann^[9]. King^[8] also treated the torque on a circular disk, as did Kotani, but took account the disk's own motion, again to several terms in λ . Wood^[10] also indicated how the torque on a circular disk is modified by its motion and checked his results by experimenting on disks in water. N. Kawai^[11] extended the calculation of the torque on a circular disk to somewhat smaller values of λ . He found that the torque reversed in sense, i.e. changed sign, when

λ became less than about .4 of the disk circumference. H. Levine^[12] obtained an expression for the force on a circular disk due to a normally incident plane wave. It is valid for long waves and is apparently also quite good for short ones. He also determined the torque on an infinitely long strip for short wavelengths^[13]. This torque oscillates around the value zero as λ varies.

The present calculation begins with two formulas for the force and torque, given without proof by H. Levine and J. Schwinger^[14]. The first of these is

$$(1) \quad F = \frac{P^2}{2\rho_0 c^2} \cos \alpha \sigma(\alpha).$$

In (1) F denotes the normal force on a disk due to an incident plane wave of (real) pressure amplitude P , α is the angle between the normal to the disk and the direction of propagation of the incident wave, $\sigma(\alpha)$ is the scattering cross section of the disk for this direction of incidence, ρ_0 is the density of the surrounding medium and c is the sound speed in it. Equation (1) applies to an infinitely long strip if F and σ are defined to be the force and scattering cross section per unit length.

The formula for the torque is, in modified notation^[14],

$$(2) \quad T = - \frac{P^2}{\rho_0 \omega c} \operatorname{Re} \{ f_{\theta}(\alpha, 0, \alpha) \}.$$

In (2) T is the torque around an axis perpendicular to the plane containing the direction of propagation of the incident plane wave and the normal to the disk. If this axis is the z -axis then the propagation direction lies in the xy -plane and the disk lies in the yz -plane. The angle between the propagation direction and the positive x -axis is denoted by α . Then $\operatorname{Pr}(\theta, 0, \alpha)$ is the complex amplitude of the wave scattered in a direction in the xy -plane making the angle θ with the positive x -axis. The angular frequency of the incident wave is ω and the torque T is positive if it tends to increase α .

To make the above definitions more precise we note that the incident pressure p_0 and the scattered pressure p_s are given by

$$(3) \quad p_0 = \operatorname{Re} \left\{ P e^{ik(x \cos \alpha + y \sin \alpha) - i\omega t} \right\}$$

$$(4) \quad p_s \sim \operatorname{Re} \left\{ P f(\phi, \chi, \alpha) \cdot \frac{-ke^{i(kr - \omega t)}}{2\pi r} \right\}$$

If the disk is an infinitely long strip parallel to the z -axis, $f_\phi(\alpha, 0, \alpha)$ in (2) must be replaced by $f_\phi(\alpha, \alpha)$ which is defined by

$$(5) \quad p_s \sim \operatorname{Re} \left\{ P f(\phi, \alpha) \right\} \cdot \frac{-\sqrt{k}}{2\pi r} e^{i(kr + \frac{\pi}{4} - \omega t)}.$$

Furthermore T then denotes torque per unit length.

Equations (1) and (2) are derived in the Appendix to this paper for both finite disks and infinite strips. The derivation is patterned after that of H. Levine, who derived (2) for the case of an infinite strip^[13] and (1) for finite disks with $\alpha = 0$ ^[12]. Equation (1) for finite disks with $\alpha = 0$ also follows from an identity derived by P.J. Westervelt^{[15], [16]}. This same identity has also been derived by O.K. Mawardi^[18] who also gives additional references to work on 'radiation pressure'.

2. The torque on a strip

The scattering amplitude $f(\phi, \chi, \alpha)$ or $f(\phi, \alpha)$ for a rigid disk is the negative of the corresponding amplitude for an aperture of the same size and shape in a soft screen. This applies in the range $0 \leq \phi \leq \frac{\pi}{2}$ and follows from the rigorous form of Babinet's principle (cf. [17], p.39). From this fact and the cross section theorem we find that $\sigma(\alpha) = 2\sigma_a(\alpha)$, where $\sigma_a(\alpha)$ is the transmission cross section of the aperture. In a previous article^[2] we have determined f and σ_a for apertures bounded by convex curves and for infinite slits in soft screens.

Therefore to calculate F for the corresponding disks we need merely insert the value of $2\sigma_a$ into (1). For a strip of width $2a$ we obtain from (27) of [2]

$$(6) F = \frac{2aP^2}{\rho_0 c^2} \left\{ \cos^2 \alpha - \frac{\cos \alpha}{2\pi^{1/2}(ka)^{3/2}} \left[\frac{\cos[2ka(1-\sin \alpha) - \frac{\pi}{4}]}{1-\sin \alpha} + \frac{\cos[2ka(1+\sin \alpha) - \frac{\pi}{4}]}{1+\sin \alpha} \right] \right\}.$$

At normal incidence ($\alpha = 0$) this becomes

$$(7) F = \frac{2a P^2}{\rho_0 c^2} \left\{ 1 - \frac{\cos(2ka - \frac{\pi}{4})}{\pi^{1/2}(ka)^{3/2}} \right\}.$$

For a disc of radius a we find from (64) of [2]

$$(8) F = \frac{\pi a^2 P^2}{\rho_0 c^2} \left\{ \cos^2 \alpha - \frac{1}{\pi(ka)^2(\sin \alpha)^{1/2}} \left[\frac{\sin[2ka(1-\sin \alpha)]}{(1-\sin \alpha)^{1/2}} - \frac{\cos[2ka(1+\sin \alpha)]}{(1+\sin \alpha)^{1/2}} \right] \right\}$$

If $\alpha = 0$ we obtain from (47) of [2], instead of (7),

$$(9) F = \frac{\pi a^2 P^2}{\rho_0 c^2} \left\{ 1 - \frac{2 \sin(2ka - \frac{\pi}{4})}{\pi^{1/2}(ka)^{3/2}} \right\}.$$

Since other formulas and some graphs of σ_a are given in [2] we will not consider F any further.

To calculate T from (2) we must compute $-\text{Re}\{f_\emptyset\}$ and evaluate it in the forward direction $\emptyset = \alpha$, $\chi = 0$. Let us first consider the case of a strip of width $2a$ with the z -axis at its center. From [2], $-f = f_s + f_m$ where f_s results from rays singly diffracted at the edges of the strip and f_m results from multiply diffracted rays. Upon using (23) and (33) of [2] and observing that due to a difference in notation the forward direction is at $\emptyset = -\alpha$, we find

$$(10) \quad \text{Re}\{f_{s\emptyset}(-\alpha, \alpha)\} = -\frac{\sin \alpha}{2k \cos^2 \alpha},$$

$$(11) \quad f_{m_g}(-\alpha, \alpha) = \left[\ln ika e^{-\ln ika} + 1 \right]^{-1} \left[k^{-1} \sec\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) \sec\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) \right. \\ \left. - 2ika \sqrt{\frac{\pi a}{k}} \left\{ e^{-i \left[2ka(1-\sin \alpha) + \frac{\pi}{4} \right]} \left[\cos \alpha - \frac{1}{2ika} \tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) \right] \sec^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) \right. \right. \right. \\ \left. \left. - e^{-i \left[2ka(1+\sin \alpha) + \frac{\pi}{4} \right]} \left[\cos \alpha - \frac{1}{2ika} \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) \right] \sec^2\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) \right\} \right].$$

From (11), retaining only the leading terms in $(ka)^{-1}$, we obtain

$$(12) \quad \operatorname{Re}\{f_{m_g}(-\alpha, \alpha)\} \sim -\left(\frac{a}{\pi k}\right)^{1/2} \frac{\cos \alpha}{2} \left[\frac{\cos[2ka(1+\sin \alpha) - \frac{\pi}{4}]}{\cos^2(\frac{\pi}{4} - \frac{\alpha}{2})} - \frac{\cos[2ka(1-\sin \alpha) - \frac{\pi}{4}]}{\cos^2(\frac{\pi}{4} + \frac{\alpha}{2})} \right].$$

According to (10), f_g contributes to T a term of order k^{-1} while (12) shows that f_m contributes a term of order $k^{-1/2}$. Therefore the main contribution to T comes from f_m and it is, according to (2) and (12),

$$(13) \quad T \sim \frac{a^2 P^2}{\rho_0 c^2} \frac{\cos \alpha}{\sqrt{\pi}(ka)^{3/2}} \left[\frac{\cos[2ka(1-\sin \alpha) - \frac{\pi}{4}]}{1 - \sin \alpha} - \frac{\cos[2ka(1+\sin \alpha) - \frac{\pi}{4}]}{1 + \sin \alpha} \right].$$

This same result, which applies for $\alpha \neq \frac{\pi}{2}$, has also been obtained by H. Levine^[13].

The first correction to it can be obtained from (10) and further corrections from (11). The result (13) can be obtained merely by taking account of the two doubly diffracted rays, rather than of all multiply diffracted ones. It is surprising that these doubly diffracted rays yield a larger contribution to the torque than do the singly diffracted rays. The torque (13) is similar in form to the second term in (6).

From (13) we see that for fixed values of α and $P^2 a^2 / \rho_0 c^2$ the torque tends to zero like $(ka)^{-3/2}$, but in an oscillatory manner. The force F , on the other hand, tends to a finite limit. Therefore for large ka the force is much

larger than the torque, which is just the reverse of the behavior at small ka . For fixed values of $P_a^2/\rho_0 c^2$ and of ka , the torque is an oscillatory odd function of α . It vanishes for certain values of α which determine the equilibrium positions of a strip which is free to rotate around the z -axis. These equilibria are stable if $T_\alpha < 0$ and unstable if $T_\alpha > 0$, so they are alternately stable and unstable. To find these values we may rewrite (13) in the form

$$(14) \quad T \sim \frac{2P_a^2 a^2}{\pi^{1/2} \rho_0 c^2 (ka)^{3/2} \cos \alpha} \left[\sin(2ka - \frac{\pi}{4}) \sin(2ka \sin \alpha) \right. \\ \left. + \sin \alpha \cos(2ka - \frac{\pi}{4}) \cos(2ka \sin \alpha) \right].$$

Thus the equilibrium values of α , other than $\alpha = 0$, are the solutions of the equation

$$(15) \quad \frac{\tan(2ka \sin \alpha)}{2ka \sin \alpha} = - \frac{\cot(2ka - \frac{\pi}{4})}{2ka}.$$

The stability of $\alpha = 0$ is determined by the sign of

$$(16) \quad T_\alpha(0) = \frac{2P_a^2 a^2}{\pi^{1/2} \rho_0 c^2 (ka)^{3/2}} \left[2ka \sin(2ka - \frac{\pi}{4}) + \cos(2ka - \frac{\pi}{4}) \right].$$

Although, as is well known, $\alpha = 0$ is stable for ka very small (when our formulas do not apply), we see that it becomes alternately unstable and stable as ka increases. When $\alpha = 0$ becomes unstable, there are two stable equilibria near $\alpha = 0$.

3. The torque on a circular disk

In order to calculate T for a disk bounded by a smooth convex curve we begin with (57) of [2]. This equation gives u_d , the field which

results when an incident field of unit amplitude is doubly diffracted at the edges of the complementary aperture in a soft screen. Therefore when the spherical wave factor is removed, this equation will yield $-f_d(\phi, \chi, \alpha)$, the contribution of doubly diffracted rays to the scattered amplitude. By using this result in (2) we will obtain the contribution of such rays to the torque. It is to be expected that, as in the case of the strip, these rays will yield the main contribution to the torque. Therefore we will not bother to evaluate the contributions of other rays.

Since the leading term in k of the derivative $f_{d\phi}$ comes from the phase factor, it suffices to evaluate the amplitude in (57) in the forward direction $\phi = \alpha$ before differentiating. Then (57) becomes (61), in which only the exponent depends upon ϕ . Upon removing the spherical wave factor we obtain

$$(17) \quad f_d(\phi, \chi, \alpha) = \frac{2}{k^2} \sum \frac{e^{ik[\Psi(P)+d+s-r]+\epsilon}}{\cos \alpha \sin \beta} \sqrt{\frac{\rho_P \rho_Q}{d|B|}}.$$

The notation is the same as that of [2]. Now we differentiate (17) and insert the result into (2), obtaining

$$(18) \quad T = \frac{2P^2}{\rho_0 \omega^2} \sum \frac{[\Psi(P)+d+s-r]_{\phi}}{\cos \alpha \sin \beta} \sqrt{\frac{\rho_P \rho_Q}{d|B|}} \sin \left\{ k[\Psi(P)+d+s-r]+\epsilon \right\}.$$

Let us apply (18) to a disk which is symmetric in the y - and z -axes. Then only two doubly diffracted rays go in the forward direction. They come from rays incident at the two points on the edge in the plane $z = 0$. Furthermore $\beta = \frac{\pi}{2}$, $\rho_P = \rho_Q = \rho$, $\Psi(P) = d_{\phi} = 0$, $s-r = \pm \frac{d}{2} \sin \phi$, $\Psi(P) = \pm \frac{d}{2} \sin \alpha$, and $B = (1 \pm \sin \alpha)(2\rho_P - d[1 \pm \sin \alpha])$. Here the positive signs apply to one ray and the negative signs to the other. Inserting these values into (18) yields

$$(19) \quad T = \frac{F^2 d^{1/2} \rho}{\rho_0 \omega^2} \left[\frac{\sin[kd(1+\sin \alpha) + \epsilon_+]}{(1+\sin \alpha)^{1/2} |2\rho-d(1+\sin \alpha)|^{1/2}} - \frac{\sin[kd(1-\sin \alpha) + \epsilon_-]}{(1-\sin \alpha)^{1/2} |2\rho-d(1-\sin \alpha)|^{1/2}} \right]$$

In (19) d is the width of the disk along the y -axis and ρ is the radius of curvature of the edge at the y -axis. The symbol $\epsilon_{\pm}$ is equal to $-\frac{\pi}{2}$ or 0 according as $2\rho - d(1 \pm \sin \alpha)$ is negative or positive.

Suppose the disk is an ellipse with major axis of length $2a$ along the y -axis and minor axis of length $2\epsilon a$ along the z -axis. Then $d = 2a$ and $\rho = \epsilon^2 a$. Since $\epsilon < 1$ it follows that $2\rho - d(1+\sin \alpha) = 2a(\epsilon^2 - 1 - \sin \alpha) < 0$, so $\epsilon_+ = -\frac{\pi}{2}$, while the value of ϵ_- depends upon the sign of $\epsilon^2 + \sin \alpha - 1$. Let us suppose that this is positive, which requires $\sin \alpha > 1 - \epsilon^2$. Then $\epsilon_- = 0$, and (19) becomes

$$(20) \quad T = -\frac{\epsilon^2 F^2 a^3}{\rho_0 c^2 (ka)^2} \left[\frac{\cos[2ka(1+\sin \alpha)]}{(1+\sin \alpha)^{1/2} (1+\sin \alpha - \epsilon^2)^{1/2}} + \frac{\sin[2ka(1-\sin \alpha)]}{(1-\sin \alpha)^{1/2} (-1+\sin \alpha + \epsilon^2)^{1/2}} \right]$$

In the special case of a circle, $\epsilon = 1$, so $\sin \alpha > 1 - \epsilon^2 = 0$ and (20) yields

$$(21) \quad T = -\frac{F^2 a^3}{\rho_0 c^2 (ka)^2 (\sin \alpha)^{1/2}} \left[\frac{\cos[2ka(1+\sin \alpha)]}{(1+\sin \alpha)^{1/2}} + \frac{\sin[2ka(1-\sin \alpha)]}{(1-\sin \alpha)^{1/2}} \right]$$

This result applies for $\alpha \neq 0, \frac{\pi}{2}$. The torque tends to zero like $(ka)^{-2}$ as ka becomes infinite, but in an oscillatory manner. For fixed ka there are equilibrium values of α which are alternately stable and unstable. They are obtained by setting $T = 0$ in (21). As in the case of the strip, the torque is larger than the force on the circular disk for ka very small, but the reverse is true for ka large.

APPENDIX: Derivation of the Basic Formulas (1) and (2)

Let us denote by $\hat{\Psi}(x, t)$ the real time dependent velocity potential of an acoustic motion at the point $x = (x, y, z)$. The pressure $p(x, t)$, to terms of second order in $\hat{\Psi}$, is given by [8]

$$(A1) \quad p = p_0 + \rho_0 \hat{\Psi}_t + \frac{\rho_0}{2} \left[c^{-2} \hat{\Psi}_t^2 - (\nabla \hat{\Psi})^2 \right].$$

Here p_0 and ρ_0 denote respectively the constant pressure and density in the undisturbed medium and c is the sound speed in it. If the motion is periodic (harmonic) in time with angular frequency ω , then $\hat{\Psi}$ can be expressed in terms of the complex time independent velocity potential $\Psi(x)$ by the relation

$$(A2) \quad \hat{\Psi}(x, t) = \text{Re } e^{-i\omega t} \Psi(x).$$

Upon inserting (A2) into (A1) and averaging over a period, we find that the average pressure $\bar{p}(x)$ is given by

$$(A3) \quad \bar{p}(x) = p_0 + \frac{\rho_0}{4} \left[k^2 \Psi \Psi^* - \nabla \Psi \cdot \nabla \Psi^* \right].$$

$$\text{In (A3) } k = \frac{\omega}{c}.$$

The time average force F in the positive x -direction on a disk S lying in the yz -plane is

$$(A4) \quad F = \int_S [\bar{p}(0-, y, z) - \bar{p}(0+, y, z)] dy dz.$$

The time average torque T around the z -axis is

$$(A5) \quad T = \int_S y [\bar{p}(0-, y, z) - \bar{p}(0+, y, z)] dy dz.$$

We will now express F and T in terms of Ψ by means of (A3). However, we first note that $\Psi_x = 0$ on the disk since it is rigid and immobile. We also make use

of the following well-known formula (cf. [17], p. 39):

$$(A6) \quad \Psi(0-, y, z) + \Psi(0+, y, z) = 2\Psi_0(0, y, z).$$

Here $\Psi_0(x)$ is the velocity potential of the incident wave, which we assume to be

$$(A7) \quad \Psi_0(x) = \frac{P}{\rho_0 \omega} e^{ik(x \cos \alpha + y \sin \alpha)}.$$

The real constant P denotes the pressure amplitude of the incident wave and α is the angle between the direction of propagation of the incident wave and the normal to the disk.

When (A3) is inserted into (A4) and (A5) certain differences occur in the integrands. By means of the preceding observations these differences can be simplified to

$$(A8) \quad (\Psi\Psi^*)_+ - (\Psi\Psi^*)_- = \operatorname{Re}(\Psi_0^* \delta\Psi),$$

$$(A9) \quad (\nabla\Psi \cdot \nabla\Psi^*)_+ - (\nabla\Psi \cdot \nabla\Psi^*)_- = \operatorname{Re}(\nabla_2\Psi_0^* \cdot \nabla_2 \delta\Psi) = \operatorname{Re}(\nabla_{0y}^* \delta\Psi_y).$$

In these formulas the subscript $+$ or $-$ indicates that a quantity is to be evaluated at $x = 0+$ or $x = 0-$. The operator $\nabla_2 = (\frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ is the gradient in the yz -plane and $\delta\Psi$ is the difference

$$(A10) \quad \delta\Psi(y, z) = \Psi(0-, y, z) - \Psi(0+, y, z).$$

When these formulas are used in (A4) and (A5) they become

$$(A11) \quad F = \frac{P}{2\omega} \operatorname{Re} \left\{ \int_S e^{-iky \sin \alpha} [k^2 \delta\Psi + ik \sin \alpha \delta\Psi_y] dy dz \right\},$$

$$(A12) \quad T = \frac{P}{2\omega} \operatorname{Re} \left\{ \int_S y e^{-iky \sin \alpha} [k^2 \delta\Psi + ik \sin \alpha \delta\Psi_y] dy dz \right\}.$$

These formulas simplify further if the second term in each integral is integrated by parts with respect to y and the fact that $\delta\mathbb{Y} = 0$ at the edge is used. They then reduce to

$$(A13) \quad F = \frac{Pk^2 \cos^2 \alpha}{2\omega} \operatorname{Re} \left\{ \int_S e^{-iky \sin \alpha} \delta\mathbb{Y} \, dydz \right\},$$

$$(A14) \quad T = \frac{P}{2\omega} \operatorname{Re} \left\{ \int_S e^{-iky \sin \alpha} \delta\mathbb{Y} [yk^2 \cos^2 \alpha - ik \sin \alpha] dydz \right\}.$$

We now make use of the fact that $\mathbb{Y}$ can be written in terms of $\delta\mathbb{Y}$ as

$$(A15) \quad \mathbb{Y}(x) = \mathbb{Y}_0(x) - \int_S \delta\mathbb{Y}(y', z') G_{\underline{x}}(x, y, z; x', y', 0) dy' dz'.$$

In (A15) G denotes the Green's function

$$(A16) \quad G(x, x') = \frac{e^{ik|x-x'|}}{4\pi|x-x'|}.$$

Let us denote the spherical coordinates of the point x by (r, θ, χ) with the x -axis as the polar axis, θ as the co-latitude and χ as the azimuth. Then for r large compared to the dimensions of the disk (A15) becomes

$$(A17) \quad \mathbb{Y}(x) \sim \mathbb{Y}_0(x) + \frac{e^{ikr}}{4\pi r} ik \cos \theta \int_S \delta\mathbb{Y}(x', y') e^{-ik(y' \sin \theta \cos \chi + z' \sin \theta \sin \chi)} dy' dz'.$$

The coefficient of $-ke^{ikr}/2\pi r$ is defined to be the amplitude of the scattered wave corresponding to the incident direction α . We write it as

$$(A18) \quad \frac{P}{p_0 \omega} f(\theta, \chi, \alpha) = -\frac{i \cos \theta}{2} \int_S \delta\mathbb{Y}(x', y') e^{-ik(y' \sin \theta \cos \chi + z' \sin \theta \sin \chi)} dy' dz'.$$

Upon comparing (A13) and (A14) with (A18) we see that F and T can be written in the form

$$(A19) \quad F = - \frac{P^2}{\rho_0 c^2} \cos \alpha \operatorname{Im} \{f(\alpha, 0, \alpha)\},$$

$$(A20) \quad T = - \frac{P^2}{\rho_0 \omega c} \operatorname{Re} \{f_\phi(\alpha, 0, \alpha)\}.$$

Equation (A20) is exactly (2) of the Introduction, while (A19) becomes (1) when we make use of the cross-section theorem^[17], $-2 \operatorname{Im} \{f(\alpha, 0, \alpha)\} = \sigma(\alpha)$.

In case the disk is a strip (or collection of parallel strips) of infinite length, the potential Ψ is independent of z . Therefore the z integration can be omitted in (A11 - A15) provided that F and T are interpreted as force and torque per unit length in the z -direction. Then G in (A15) is the two-dimensional Green's function

$$(A21) \quad G(x, x') = \frac{1}{4} H_0^{(1)}(k|x-x'|).$$

Now we denote by r, ϕ the polar coordinates of x , assumed to lie in the plane $z = 0$. When r is large compared to the strip width, (A15) becomes

$$(A22) \quad \Psi(x) \sim \Psi_0(x) + \frac{1}{4} \left(\frac{2}{\pi k r} \right)^{1/2} e^{i(kr - \frac{\pi}{4})} i k \cos \phi \int_S \delta \Psi(y') e^{-iky' \sin \phi} dy'.$$

The coefficient of $-(k/2\pi r)^{1/2} e^{i(kr - \frac{\pi}{4})}$ is defined to be the amplitude of the scattered wave. We write it as

$$(A23) \quad \frac{P}{\rho_0 \omega} f(\phi, \alpha) = - \frac{i \cos \phi}{2} \int_S \delta \Psi(y') e^{-iky' \sin \phi} dy'.$$

When (A13) and (A14), omitting the z integration, are compared with (A23),

they are found to be of the form (A19) and (A20) with $f(\alpha, \alpha)$ and $f_{\theta}(\alpha, \alpha)$ in place of $f(\alpha, 0, \alpha)$ and $f_{\theta}(\alpha, 0, \alpha)$.

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Figure 1

A disk lying in the yz -plane. The direction of propagation of the incident wave lies in the xy -plane and makes the angle α with the x -axis.

Figure 2

The torque per unit length T on a strip of width $2a$ as a function of the angle of incidence α for $ka = 5$. The vertical scale is the value of $T \left[\frac{a^2 p^2}{\rho_0 c^2 \pi^{1/2} (ka)^{3/2}} \right]^{-1}$. The curve is based upon equation (13).

The values of α at which $T = 0$ are equilibrium angles. Those for which $T_\alpha < 0$ are stable and those for which $T_\alpha > 0$ are unstable. The curve does not apply near $\alpha = \frac{\pi}{2}$.

Figure 3

The torque T on a circular disk of radius a as a function of the angle of incidence α for $ka = 5$. The vertical scale is the value of

$$\tau \left[\frac{\pi^2 a^3}{\rho_0 c^2 (ka)^2} \right]^{-1}$$

The curve is based upon equation (21). The values of α at which $T = 0$ are equilibrium angles. Those for which $T_\alpha < 0$ are stable and those for which $T_\alpha > 0$ are unstable. The curve does not apply near $\alpha = 0$ and $\alpha = \pi/2$.

Figure 4

The torque per unit length T on a strip of width $2a$ as a function of ka for $\alpha = \pi/4 (45^\circ)$. The vertical scale is the value of

$$\tau \left[\frac{\pi^2 a^3}{\rho_0 c^2 (ka)^2} \right]^{-1}$$

The curve is based upon equation (13). The values of α at which $T = 0$ are equilibrium angles. Those for which $T_\alpha < 0$ are stable and those for which $T_\alpha > 0$ are unstable. The curve does not apply for ka small.

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